First Terminal Examination 2015 - 2016

Class – XII

Subject – Mathematics

Time : 3 Hours | **Max. Marks : 100**

General Instructions :

(a) All questions are compulsory.

(b) The question paper consists of 26 questions divided into 3 Sections A, B and C. Section A comprises of 6 questions of 1 mark each, Section B comprises of 13 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.

(c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(d) Do all the questions, however internal choice is provided in 4 questions of 4 marks and 2 questions of 6 marks.

(e) Use of calculators is not permitted.

SECTION – 'A' (1×6=6)

1. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$.

2. If $= \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$, then find $|\text{adj } A|$. $-9-2$ (-11)

3. Differentiate $\cos x^{o}$ w.r.t x.

4. Find the equation of normal to curve $y = 2x^2 + 3\sin x$ at $x = 0$.

5. Integrate : $\int \frac{\sin x}{1+\sin x}\,dx$ $x = 3x$.

6. Evaluate : $\int_{0.5}^{2} [x]\,dx$

SECTION – 'B' (4×13=52)

7. Write in simplest form :

$$\cos^{-1}\left[\frac{3}{5}\cos x + \frac{4}{5}\sin x\right]$$

8. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$,

prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

9. Prove that any square matrix can be uniquely expressed as a sum of a symmetric matrix and a skew symmetric matrix.

10. In a triangle ABC, if $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$, then

prove that triangle ABC is an isosceles triangle.

OR

$4x - 2x - 3$

$1 - \cos^2 x + \sin^2$

$2\sin^2\theta$

Show that if the determinant $\begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$ then $\sin\theta = 0$ or $\frac{1}{2}$.

11. Find the value of 'a' for which the function 'f' defined as

$6x - 2$

$$f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1) & , x \le 0 \\ \dfrac{\tan x - \sin x}{x^3} & , x > 0 \end{cases}$$ is continuous at $x = 0$.

$x = \cos\alpha$

$y = \cos\beta$

12. Find $\frac{dy}{dx}$ if $y = x^{1+\frac{1}{x}} + \left(x + \frac{1}{x}\right)^x$

$\cos(\alpha+\beta) = xy + \sqrt{1-x^2}\sqrt{1-y^2}$

$+\sin^{-1}(z)$

OR

Find $\frac{dy}{dx}$ if $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$

$\cos\left(\frac{\pi}{2}\right.$

$-\sin^{-1}(x)$

13. Show that curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

14. Find the intervals in which the function defined as $f(X) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing and decreasing.

OR

Show that $\dfrac{x}{1+x} < \log(1 + x) < x$ for $x > 0$.

$\sin 3\theta = \sin(\theta + 2\theta)$
$= \sin\theta(\cos 2\theta) + \cos\theta \sin 2\theta$
$= \sin\theta(1 - 2\sin^2\theta) + 2\sin\theta\cos^2\theta$
$= \sin\theta - 2\sin^3\theta + 2\sin\theta(1 - \sin^2\theta)$
$= \sin\theta - 2\sin^3\theta + 2\sin\theta - 2\sin^3\theta$
$= 3\sin\theta - 4\sin^3\theta$

15. Evaluate : $\displaystyle\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \alpha)}}$.

$1 + x = e^y$
$x = e^y - 1$

16. Integrate : $\displaystyle\int e^x\left(\frac{2 + \sin 2x}{1 + \cos 2x}\right) dx$

$1 - \cos 2\theta = 2\sin^2\theta$

OR

Evaluate : $\displaystyle\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

17. Evaluate : $\displaystyle\int_0^{\frac{\pi}{2}} \log \sin x \, dx$

18. Evaluate : $\displaystyle\int_{-1}^{2} |x^3 - x| \, dx$

19. Using integration find the area of region bounded by line $2y = -x + 8$, x-axis and lines $x = 2$ and $x = 4$.

$x^3 - x < 0$
$x(x^2 - 1) < 0$

SECTION – 'C' (6×7=42)

20. Let 'X' be a non empty set and P(X) be its power set. Let $*$ be an operation defined on elements of P(X), by $A * B = A \cap B$ for all A, B $\in$ X. Then :

(a) Prove that $*$ is a binary operation on P(X).

(b) Is $*$ commutative ?

(c) Is $*$ associative ?

(d) Find identity element in P(X) w.r.t. $*$.

$A \cap B \cap C$

13. Show that curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

14. Find the intervals in which the function defined as $f(X) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing and decreasing.

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Show that $\dfrac{x}{1+x} < \log(1 + x) < x$ for $x > 0$.

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$\sin 3\theta = \sin(\theta + 2\theta)$
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$= \sin\theta(1 - 2\sin^2\theta) + 2\sin\theta(1 - \sin^2\theta)$
$= \sin\theta - 2\sin^3\theta + 2\sin\theta - 2\sin^3\theta$
$= 3\sin\theta - 4\sin^3\theta$

$1 + x = e^y$
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(b) Is $*$ commutative ?

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(d) Find identity element in P(X) w.r.t. $*$.

$A \cap B \cap C$

(e) Find all invertible elements of P(X).

(f) If 'o' is another binary operation defined on P(X) as AOB = A ∪ B then verify that 'o' distributes itself over ∗

21. Find the inverse of the following matrix, if exists, using elementary transformations :

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

OR

Solve following system of equations using matrices :

$$2x - y + z = 4$$
$$x + 3y + 2z = 12$$
$$3x + 2y + 3z = 16$$

$t^2\left(1+\frac{1}{t^2}\right)$

22. Find the area of greatest rectangle that can be inscribed in an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

a cosθ

OR

Prove that the least perimeter of an isosceles triangle in which a circle of radius 'r' can be inscribed is $6r\sqrt{3}$.

23. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 0 and y = 4 in to three equal parts.

24. Differentiate $\tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$ w.r.t. $\cos^{-1} 2x\sqrt{1-x^2}$ when $x \left(\frac{1}{\sqrt{2}}, 1\right)$.

25. A manufacturer makes two types of machines. Deluxe sells for ₹ 12,000 and standard sells for ₹ 8000. It costs ₹ 9,000 to produce a deluxe and ₹ 6000 to produce a standard. In one week, manufacturer can produce 40 to 60 deluxe machines and 30 to 50 standard machines but not more than 90 machines. Formulate the problem as LPP and find the number of standard and deluxe machines to be made to have maximum profit.

26. Evaluate : $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

(A∪B)∪C

A∪C ∩ B∪C

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