St Anthony Sr Sec School Jamnagar

HALF YEARLY EXAMINATION -2016

XII MATHEMATICS

(Set I)

Time : 3 Hours M.M.100

SECTION A (*Questions 1 – 4 carry 1 mark each*)

1. If $*$ is a binary operation on the set R defined by $a * b = a + b + 5$. Find its identity element.

2. If $\begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$, then write the value of $(x + y + z)$

3. If $y = cot^{-1}x + cot^{-1}\left(\frac{1}{x}\right)$ Write $\frac{dy}{dx}$ in simplest form.

4. Evaluate : $\int_{-4}^{-2} \frac{1}{x}\, dx$

SECTION B (*Questions 5– 12 carry 2 mark each*)

5. If $f: X \to Y$ is invertible function, show that f has unique inverse.

6. Prove that : $tan^{-1}x + tan^{-1}(x + 1) = tan^{-1}(x^2 + x + 1)$

7. If $cos(2sin^{-1}x) = \frac{1}{9}$, then find the value of x.

8. Find the value of x such that $\begin{bmatrix} 1 & 1 & x \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$

9. Using $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ show that $(AB)' = B'A'$

10. Using properties of determinants, show that the points $(a, (b + c)), (b, (c + a))$ and $(c, (a + b))$ are collinear.

11. Find $\frac{dy}{dx}$ if $3\sin(xy) + 4\cos(xy) = 5$

12. Divide the interval $\left(0, \frac{\pi}{2}\right)$ into sub intervals in which the function $f(x) = tan^{-1}(\sin x + \cos x)$ is increasing or decreasing.

SECTIONC (*Questions 13– 23 carry 4 mark each*)

13. Let the function $f: R \to R$ be defined by $f(x) = \cos x$. give reasons why is $f(x)$ neither one – one nor on to.

14. Prove that $tan^{-1}1 + tan^{-1}2 + tan^{-1}3 = \pi$

15. Express the matrix $A = \begin{bmatrix} 2 & 5 \\ 7 & 6 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrices.

16. Find the value $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ where A_{ij} is the cofactor of the element a_{ij} of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \\ 5 & 9 & 3 \end{bmatrix}$

12 +10 22 27 -22 5

17. If $\begin{vmatrix} a & b+y & c+z \\ a+x & b & c+z \\ a+x & b+y & c \end{vmatrix} = 0$, prove that $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} + 2 = 0$ (using properties)

18. If $y = 3\cos(\log x) + 4\sin(\log x)$, then prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

19. Check the continuity and differentiability of the function $f(x)$ at $x = 2$.

$$f(x) = \begin{cases} 1 + x, & \text{when } x \leq 2 \\ 5 - x, & \text{when } x > 2 \end{cases}$$

20. A man 160 cm tall is walking away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1 m/sec. How fast is the tip of his shadow moving. (consider the length as well as displacement of the shadow)

21. At what points will the tangents to the curve $y = 2x^3 - 15x^2 + 36x - 21$ be parrallel to the line $y = 5$. Also find the equations of those tangents.

22. Evaluate the integral $\int \frac{1}{9x^2 + 6x + 5}\, dx$.

23. $\int_2^8 f(x)\, dx$, where $f(x) = |x - 2| + |x - 3| + |x - 4|$

SECTION D (*Questions 24 – 29 carry 6 mark each*)

24. Solve the following system of equations using the inverse o a matrix.

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4 \qquad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \qquad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

25. A closed right circular cylinder has a volume of 2156 cm^3. What will be the radius of its base so that its total surface area is minimum ?

26. If the sum of hypotenuse and a side of a right triangle is a constant, show that the area of the triangle is maximum, when the angle between them is 60^0.

27. Evaluate as the limit of a sum: $\int_2^5 (2x^2 - x + 5)\, dx$

28. Find the area of the region bounded by $y^2 \leq 8x$ and $x^2 + y^2 = 9$.

29. A co – operative society of farmers has 50 hectars of land to grow Rice and Wheat. The profits from Rice and Wheat are estimated to be ₹10,500 and ₹ 9,000 per hectar respectively. To control weeds, a liquid herbiside has to be used for these crops at the rate 20 litres and 10 litres per hectar respectively. Further, not more than 800 litres of the herbiside to be used in order to protect fish in the pond which collect drainage from this land. How much land should be allocated to ech of these crops so as to maximise the profit ?

Also discuss : 'herbiside is boon or bane'.